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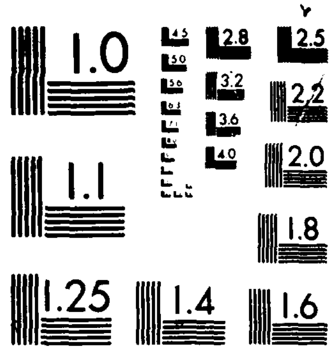

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An Analytical Soil Thermodynamic Model for the Diurnal Variation of Ground Surface Temperature

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5 January 1987

ATMOSPHERIC SCIENCES DIVISION PROJECT 6670

AIR FORCE GEOPHYSICS LABORATORY

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"This technical report has been reviewed and is approved for publication"

FOR THE COMMANDER

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An analytical expression is given for time variations of temperature within a model soil slab that is subjected to idealized heating at the upper surface and maintained at constant temperature at the lower surface. Applied at the upper surface, the model temperature may be considered to be the ground surface temperature of the earth. An important feature of this model is that phase-shifted non-linear responses to the forcing are allowed at the upper boundary. The analytical nature of the solution enables us to easily study the relative importance of various physical processes in the model surface energy budget equation. For example, evaporational cooling is found to be an important feedback process under model summer conditions, accounting for a change of about 6K in the daily mean surface temperature; under winter conditions, the change attributable to evaporation is only about 1K. The perturbation method used in our approach to linearize the model upper boundary condition is shown to be applicable to more complicated and more realistic upper boundary conditions.

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1. INTRODUCTION

The determination of diurnal variations of ground surface temperature plays a critical role in the modeling of the atmospheric planetary boundary layer and, by extension, in mesoscale numerical weather prediction (NWP) models. In theory, the problem is a relatively simple one because, to a very good approximation, these variations may be represented by a linear partial differential equation. Assuming homogeneity and constancy in thermal characteristics of a soil slab that is subject to linear forcing at the bounding surfaces, we can easily solve the linear heat equation analytically. Ground surface temperature may then be obtained by applying the solution at the upper surface of the slab. Solutions of linear flow of heat in a slab bounded by two parallel planes have been given, for example, by Carslaw and Jaeger[1] for various kinds of linear forcing at the bounding surfaces. In geophysics, Campbell[2] has given a time-dependent solution of the temperature distribution in an infinite soil layer that is forced at the upper surface by a peri-

(Received for publication 18 December 1986)

1. Carslaw, H.S., and Jaeger, J.C. (1959) Conduction of Heat in Solids. Oxford University Press, London, 510 pp.

2. Campbell, Gaylon S. (1977) An Introduction to Environmental Biophysics. Springer-Verlage, 159 pp.

odic temperature wave. Blackadar[3] has provided a solution for a soil model of infinite depth, subject to the boundary conditions that (1) the top of the soil is in instantaneous thermal equilibrium and is heated by fluxes that are represented by a single-component cosine function in time; (2) the temperature at infinite depth approaches a constant value T_d.

At the ground surface, the various flux terms of the energy balance equation unfortunately are not, in general, linear functions of T_g, the temperature at the ground. These terms include: short wave flux due to solar radiation, infrared flux from the atmosphere, thermal emission at the ground, sensible heat transfer between the air and the ground, latent heat flux due to evaporation/condensation and rainfall, and soil heat flux due to the vertical thermal gradient within the soil. Of these, only the first has well defined, externally forced diurnal and seasonal periods and then only under cloudless skies. Therefore, the difficulty in obtaining an analytical solution for the heat equation lies not in the differential equation itself but rather in accurately prescribing the external forcing and in the nonlinear feedback nature of the upper boundary condition. For this reason, values of T_g in NWP-type models are usually determined by numerical methods, Deardorff[4] found the so-called force-restore method to be the most satisfactory. This method is currently used in many NWP-type models.[5, 6]

In this report, we present a perturbation method that may be used to obtain approximate T_g values analytically for a soil-slab model. The key is to decompose T_g into a basic part and a perturbed part and then approximate all surface flux terms other than those due to solar heating as linear functions of the perturbed temperature. Once such approximations are made, our problem then becomes that of linear heat flow in a finite soil slab subject to boundary conditions that are linear functions of the perturbed temperature. Such a problem may be solved analytically. T_g is then obtained simply by applying the solution at the upper surface of the slab. This model, which permits phase shifts between forcing and feedback processes at the upper boundary, may be viewed as a refined version of the classical soil model described by Blackadar.[3] A similar model has been solved numeri-

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3. Blackadar, A.K. (1979) High resolution models of the planetary boundary layer. Advances in Environmental Science and Engineering, 1, No. 1, J. Pfafflin and E. Ziegler, Eds., Gordon and Breach, 50-85.

4. Deardorff, J.W. (1978) Efficient prediction of ground surface temperature and moisture, with inclusion of a layer of vegetation. J. Geophys. Res., 83 (No. C4):1889-1903.

5. Zhang, D., and Anthes, R.A. (1982) A high-resolution model of the planetary layer--sensitivity tests and comparison with SESAME--79 data. J. Appl. Meteorol., 21:1594-1609.

6. Tuccillo, J.J., and Phillips, N.A. (1986) Modeling of Physical Processes in Nested Grid Model. Technical Procedures Bulletin No. 36.3, National Weather Service, Silver Spring, Md.

cally in a multi-layer setting by Cautenet et al.[7] for a comparison with experimental data in the African savannah. In addition to describing temperature variations within a soil slab analytically, our model also enables us to determine explicitly the soil flux term within the slab, a term that plays a central role in the force-restore method. Furthermore, the perturbation method adopted here also points the way for linearizing models with more complicated time-dependent forcing at the upper boundary.

2. MODEL EQUATIONS

The variation of temperature T in a soil slab is given as a function of time t and depth z by the heat equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \qquad d \leq z \leq 0 \tag{1}$$

where κ is the thermal diffusivity of the soil. We attempt to solve this heat equation analytically subject to the mixed boundary conditions:

(i) Energy balance is maintained at the upper surface ($z = 0$),

$$\lambda \frac{\partial T}{\partial z} = I_s + I_L - I_g - H_s - LE; \tag{2}$$

(ii) Temperature at the bottom of the slab ($z = d$) is constant,

$$T = T_d. \tag{3}$$

Here, surface energy balance among the various flux terms mentioned in Section 1 is represented symbolically by Eq. (2). These and other symbols are defined in the Glossary. In this model, we have made the following assumptions:

(1) The thermal diffusivity of the soil κ is constant in time and throughout the slab.

(2) There is a balance of all heat fluxes at the ground surface; temperature at a depth d is maintained at a constant value T_d.

(3) The initial condition satisfied the equilibrium solution for the given forcing at the boundaries.

(4) The atmospheric transmissivity for short wave radiation may be approximated by a mean value $\bar{\tau}$.

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7. Cautenet, Y., Coulibaly, Y., and Boutin, C. (1985) Calculation of ground temperature and fluxes by surface models: A comparison with experimental data in the African savannah. Tellus, 37B, 64-77.

We shall assume further that:

(5) The solar flux I_s in Eq. (2) may be decomposed into a daily mean $\bar{I}$ and a diurnal variation $I' \cos \omega t$. Thus

$$I_s = \bar{I} + I' \cos \omega t$$

where

$$\bar{I} = [\omega_s a_0 + a_1 \sin \omega_s] / \pi \quad \text{(Ref. 8)}$$

$$I' = (a_0 + a_1) - \bar{I}$$

$\omega_s = \cos^{-1} [-\tan \phi \tan \delta]$ is the local sunrise hour-angle in radians.

(6) Fluxes other than solar flux may be parameterized in the following manner:

$$I_L = \varepsilon_a \sigma T_a^4$$
$$I_g = \varepsilon \sigma T_g^4$$
$$H_s = C_p \rho_a C_{HO} u_a (T_g - T_a)$$
$$LE = L \rho_a C_{HO} u_a A_m [q_s(T_g) - q_a]$$

(7) The parameters u_a, T_a, q_a, and A_m are externally specified.

We emphasize here that in this model:

(1) Phase shifts in the feedback mechanism are permitted through terms containing T_g in Eq. (2);

(2) The flux terms I_g and LE are nonlinear functions of T_g.

3. LINEARIZATION OF THE UPPER BOUNDARY CONDITION

As explained above, the nonlinearity in our model is due to the upper boundary condition, Eq. (2). Attempts have been made in the past to linearize the surface energy equation. For example, Deardorff[4] made the equation linear by expanding T_g and $q_s(T_g)$ about a given instant in time (time-step n). Pan and Mahrt[9] expanded T in z and the potential evaporation, E_p, in time to get a linear equation. We shall

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8. Iqbal, M. (1983) An Introduction to Solar Radiation. Academic Press, Toronto, Canada, 390 pp.

9. Pan, H.L., and Mahrt, L. (1987) Interaction between soil hydrology and boundary-layer development. Boundary Layer Meteorol., 38:185-202.

linearize Eq. (2) by expanding T_g and q_s about their respective daily mean values at the ground surface. Thus we write

$$T = \overline{T}_g + D(z, t), \tag{4}$$

where $\overline{T}_g$ is the as-yet undetermined daily mean temperature at the ground, and D is the deviation from $\overline{T}_g$. Since $\overline{T}_g >> D$, we may make the approximation

$$T_g^4 = (\overline{T}_g + D)^4 \simeq \overline{T}_g^4 + 4\overline{T}_g^3 D. \tag{5a}$$

Assuming that the diurnal amplitude of T_a is 1/3 that of T_g, we write

$$T_a^4 \simeq \overline{T}_a^4 + 4\overline{T}_a^3 D/3. \tag{5b}$$

Next, we expand $q_s(T)$ about $q_s(\overline{T})$:

$$q_s(T) = q_s(\overline{T}) + C\left(\frac{\partial q_s}{\partial T}\right)_{\overline{T}} D, \tag{6}$$

where T represents either T_g or T_a; and C = 1 when $\overline{T} = \overline{T}_g$, and C = 1/3 when $\overline{T} = \overline{T}_a$. Here $(\partial q_s/\partial T)_{\overline{T}}$ is given by the Clausius-Clapeyron equation

$$\left(\frac{\partial q_s}{\partial T}\right)_{\overline{T}} = \frac{m_v L}{R^*} \frac{q_s(\overline{T})}{\overline{T}^2} , \tag{7}$$

and $q_s(\overline{T})$ is given by

$$q_s(\overline{T}) = (380/p_*) \exp [17.27 (\overline{T} - 273.16)/(\overline{T} - 35.86)],$$

where m_v is the molecular weight of water vapor, p_* is the surface pressure and R^* is the universal gas constant. Using these approximations, we may now write Eq. (2) as a linear function in D:

$$\begin{aligned} \lambda \frac{\partial D}{\partial z} = {} & \overline{I} + I' \cos \omega t + \varepsilon_a \sigma (\overline{T}_a^4 + 4\overline{T}_a^3 D/3) \\ & - \varepsilon \sigma (\overline{T}_g^4 + 4\overline{T}^3 D) - \gamma C_p (\overline{T}_g - \overline{T}_a + 2D/3) \\ & - \gamma L A_m [q_s(\overline{T}_g) - 0.5 q_s(\overline{T}_a)] \\ & - \gamma L A_m \left[\left(\frac{\partial q_s}{\partial T}\right)_{\overline{T}_g} - \frac{0.5}{3}\left(\frac{\partial q_s}{\partial \overline{T}}\right)_{\overline{T}_a}\right] D, \end{aligned} \tag{8}$$

where $\gamma = \rho_a u_a C_{HO}$. We have assumed here that $q(T_a) = 0.5\, q_s(\overline{T}_a)$, that is, the relative humidity of the air is 50 percent.

1. LINEARIZED MODEL SOLUTION

With the help of Eq. (4), we write model Eqs. (1) through (3) in terms of D:

$$\frac{\partial D}{\partial t} = \kappa \frac{\partial^2 D}{\partial z^2}, \qquad d \leq z \leq 0, \tag{9}$$

$$\lambda \frac{\partial D}{\partial z} = \overline{F} + I' \cos \omega t - bD, \qquad z = 0 \tag{10}$$

$$D = T_d - \overline{T}_g, \qquad z = d, \tag{11}$$

where

$$\overline{F} = \overline{I} + \overline{I}_L - \epsilon \sigma \overline{T}_g^4 - \gamma C_p (\overline{T}_g - \overline{T}_a) - \overline{LE}, \tag{12a}$$

$$b = 4\sigma(\epsilon \overline{T}_g^3 - \epsilon_a \overline{T}_a^3/3) + 2\gamma C_p/3 + \gamma L A_m \left[\left(\frac{\partial q_s}{\partial t} \right)_{\overline{T}_g} - \frac{0.5}{3} \left(\frac{\partial q_s}{\partial T} \right)_{\overline{T}_g} \right], \tag{12b}$$

$$\overline{I}_L = \epsilon_a \sigma \overline{T}_a^4, \tag{12c}$$

$$\overline{LE} = \gamma L A_m [q_s(\overline{T}_g) - q(\overline{T}_a)]. \tag{12d}$$

Thus, in terms of the new variable D, the balance of heat fluxes at the upper boundary is among a soil flux term, a time-mean flux term $\overline{F}$, a time-dependent term $I' \cos \omega t$, and a feedback term bD. This upper boundary condition is linear in D provided that $\overline{T}_g$ is known. It will be seen later in Eq. (20b) that the value of $\overline{T}_g$ is determined by the balance of mean fluxes. Written in the form of Eqs. (12a) and (12b), the various physical processes contributing to the forcing and the feedback terms can easily be isolated. We shall illustrate the relative importance of some of these terms with numerical examples.

The linear model Eq. (9) has a solution of the form

$$D = \tilde{D} e^{\beta z} \cos(\omega t - \alpha + \beta z) + [C_0 + \tilde{D} C_1 \cos(\omega t - \alpha + \beta d)] z, \tag{13}$$

where the constants α, β, $\tilde{D}$, C_1, and C_0 must satisfy both the differential equa-

tion and the boundary conditions. Note that the time-independent component $C_0 z$ is attributable to the mean flux term $\overline{F}$ in the boundary condition. To determine these constants, we first substitute the differentiated form of Eq. (13) into Eq. (9) to yield

$$\beta = (\omega / 2\kappa)^{1/2}. \tag{14}$$

Next, substituting for D in the lower boundary condition Eq. (11) and evaluating at $z = d$, we have

$$C_0 d + \tilde{D}\,(C_1 d + e^{\beta d}) \cos(\omega t - \alpha + \beta d) = T_d - \overline{T}_g. \tag{15}$$

Equating coefficients for the time-dependent and time-independent parts, respectively, we get

$$C_0 = (T_d - \overline{T}_g)/d, \tag{16}$$

$$C_1 = -\exp(2\beta d). \tag{17}$$

Finally, we substitute for D and its derivatives in Eq. (10) at the upper boundary $z = 0$ to obtain

$$\alpha = \tan^{-1}[S_1/(b + S_2)], \tag{18}$$

$$\tilde{D} = I'/[(b + S_2)\cos\alpha + S_1 \sin\alpha], \tag{19}$$

$$C_0 = \overline{F}/\lambda, \tag{20a}$$

where $S_1 = \lambda(\beta + C_1 \sin\beta d)$, $S_2 = \lambda(\beta + C_1 \cos\beta d)$. Thus, Eq. (13) is a solution to Eq. (9) provided that Eq. (20a) is satisfied and the constants are given by Eqs. (14) through (19). With the help of Eqs. (12a) and (16), Eq. (20a) may be easily identified as the daily mean surface energy-balance equation

$$\lambda(T_d - \overline{T}_g)/d = \overline{I} + \overline{I}_L - \varepsilon\sigma\overline{T}_g^{\,4} - \gamma C_p(\overline{T}_g - \overline{T}_a) - \overline{LE}. \tag{20b}$$

Thus, $\overline{T}_g$ is determined by the balance of daily mean fluxes at the ground surface. Once $\overline{T}_g$ is known, $T(z, t)$ is given by Eq. (4). In the numerical examples given in the next section, we obtained approximate solutions to Eq. (20b) by first computing $q_s(\overline{T}_g)$ in $\overline{LE}$ with a guess value for $\overline{T}_g$ and rearranging Eq. (20b) into a

quartic equation of the form

$$a\,\overline{T}_g^{\,4} + b\overline{T}_g + c = 0,$$

where

$$a = \varepsilon\sigma,$$

$$b = \gamma C_p - \lambda/d,$$

$$c = \lambda T_d/d - \overline{I} - \varepsilon_a \sigma \overline{T}_a^{\,4} - \gamma C_p \overline{T}_a + \overline{LE}.$$

The solution was obtained by using IMSL routines and then choosing the appropriate root.

At the ground surface where $z = 0$, Eq. (13) reduces to

$$D_g = \tilde{D} \cos(\omega t - \alpha). \tag{21}$$

Therefore, the temperature at the ground surface is given by

$$T_g = \overline{T}_g + \tilde{D} \cos(\omega t - \alpha). \tag{22}$$

We now see that $\tilde{D}$ is the diurnal amplitude and α is the phase lag at the ground surface.

To delineate the effect of phase shifts in the feedback terms in the upper boundary condition, we give here a solution for a model in which such phase shifts are not permitted. In this case, the upper boundary condition [Eq. (10)] becomes

$$\lambda \frac{\partial D}{\partial z} = \overline{I} + I' \cos\omega t - b\tilde{D} \cos\omega t. \tag{10a}$$

With this change in the upper boundary condition, the only changes in our earlier model solution are in Eqs. (18) and (19), which become

$$\alpha = \tan^{-1} [S_1/S_2], \tag{18a}$$

$$\tilde{D} = I'/[b + S_2 \cos\alpha + S_1 \sin\alpha]. \tag{19a}$$

On comparing these equations with Eqs. (18) and (19), it is apparent that phase shifts in the feedback terms tend to reduce the phase lag but increase the amplitude of the diurnal changes. Since, as we shall see later, $\beta >> C_1$, the phase lag given in Eq. (18a) is about 45°.

5. NUMERICAL EXAMPLES

To get a feel for the model solution of Eq. (13), we present here numerical examples for two typical cases, a summer case and a winter case, both at $\phi = 35°$. Input values for the physical constants are given in Table 1; values of the modeling parameters for the two cases are given in Tables 2a and 2b. Solutions are plotted as functions of time for several values of z in Figures 1 and 2. For the summer case in Figure 1, we see that at the ground surface the amplitude for the diurnal temperature wave is about 12.5 K, with the maximum lagging local noon by about an hour. The amplitude decreases and the phase lag increases with depth. The wave is completely damped at the bottom of the slab to a constant value of about 295 K, which is dictated by the lower boundary condition. On solving Eq. (20b), we found a value of 300 K for $\overline{T}_g$. It is interesting to see that at a depth of about 60 cm, the daily temperature minimum occurs at the time of the surface temperature maximum. For the winter case, variations with local time and depth are given in Figure 2. Compared with the summer case, the amplitudes are smaller and the phase lags are slightly larger. At the surface, we have $\overline{T}_g = 277$ K, $\tilde{D} =$ 9.5 K and a phase lag of about 1.2 hr.

To check the realism of our model solution, we reproduced in Figure 3 some observed data cited by Kuo.[10] Comparison of Figures 1 and 3 indicates that this model is capable of simulating the salient features of the observed variations during the day both in amplitude and in phase. At night, the amplitudes are some-

Table 1. Input Numerical Constants

$S = 1370$ W m^{-2}	$\kappa = 1.0 \times 10^{-6}$ m^2 sec^{-1}
$\overline{\tau} = 0.9$	$\lambda = 2.2$ J sec^{-1} m^{-1} K^{-1}
$A = 0.2$	$C_p = 1006$ J kg^{-1} K^{-1}
$\varepsilon = 0.95$	$L = 4187(597.3 + 0.566\ T)$ J kg^{-1}
$\varepsilon_a = 0.6$ (clear sky)	$p_* = 101.3$ kPa
$P = 86400$ sec	$R^* = 287$ J K^{-1}
*guess values, used in Eq. (12d) only	

10. Kuo, H.L. (1968) The thermal interaction between the atmosphere and the earth and propagation of diurnal temperature waves. J. Atmos. Sci., 25:682-706.

Table 2a. Input Values for Model Parameters

	Summer	Winter
ϕ (degrees)	35	35
δ (degrees)	21.5	-21.5
d (meters)	0.8	0.8
T_d (K)	295	274
$\overline{T}_a$ (K)	299	278
$\overline{T}_g$ (K)	301*	277*
u_a (m s^{-1})	3	4
q_a (kg kg^{-1})	$0.5\ q_s\ (\overline{T}_g)$	$0.5\ q_s\ (\overline{T}_g)$
C_{HO}	5.0×10^{-3}	5.0×10^{-3}
A_m	0.2	0.2
$q_s(\overline{T}_g)$ (kg kg^{-1})	0.023	0.005
* guess values, used in Eq. (12d) only		

Table 2b. Calculated Constants for Summer and Winter Cases

	Summer	Winter
a_0 (W m^{-2})	207	-207
a_1 (W m^{-2})	752	752
ω_s (rad)	1.85	1.29
$\overline{I}$ (W m^{-2})	352	145
I' (W m^{-2})	607	400
C_0 (K m^{-1})	13.6	5.58
C_1 (m^{-1})	0.01	0.01
β (m^{-1})	6.03	6.03
$\partial q_s / \partial T$ (T^{-1})	1.71×10^{-1}	4.35×10^{-2}

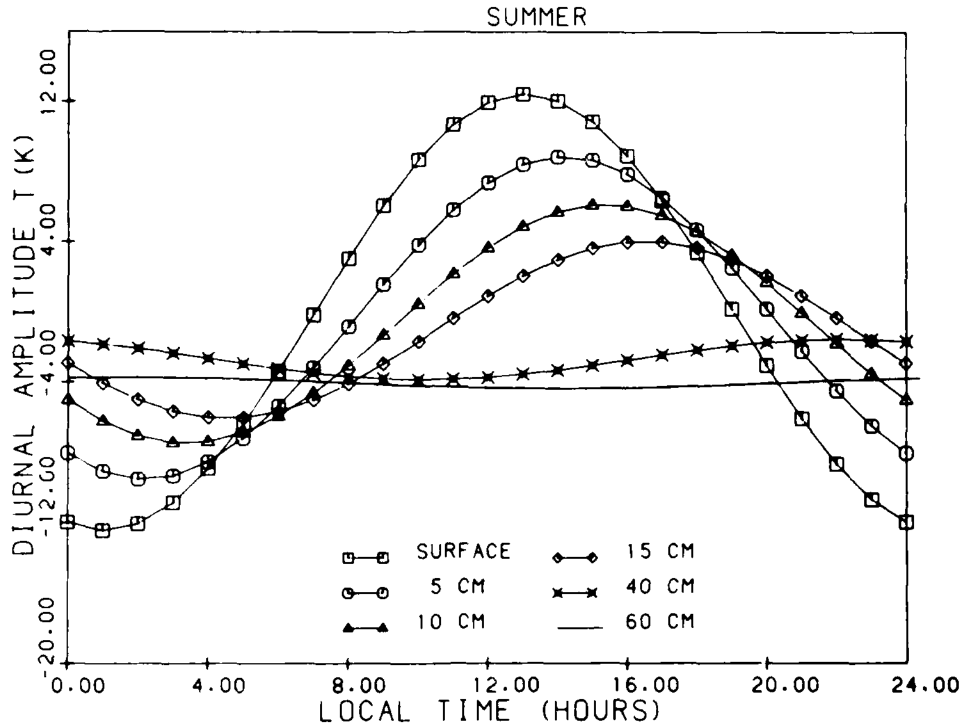

Figure 1. Model Diurnal Temperature Deviation (From the Surface Daily Mean) at Various Depths (Summer Case). Solar midnight is represented as 0.00 local time

what in error, due undoubtedly to the fact that in our model, forcing at the upper boundary is constrained to a single-component cosine wave.

A chief merit of linear models is the extreme ease with which one may gain insight into various physical processes. For example, the relative magnitude of each term in the mean surface-energy-balance equation, Eq. (20b), may be used to assess the relative importance of various heat transfer processes at the ground. Such a comparison is given in Table 3. Here the dominant fluxes are seen to be, as is well known, radiative fluxes due to the sun, the atmosphere, and the ground. For typical summer conditions, daily mean heat transfer due to nonradiational processes is small, although for the winter case where $\overline{T}_a > \overline{T}_g$, sensible heat is transferred from the atmosphere to the ground.

As another example, we examine the roles of the time-dependent terms in the upper boundary condition, Eq. (10). This can be done by examining Eqs. (18) and (19). Since C_1 as shown in Table 2b is typically three orders of magnitude smaller than b, we may write Eqs. (18) and (19) approximately as

$$\alpha = \tan^{-1} [\lambda\beta / (b + \lambda\beta)], \tag{18b}$$

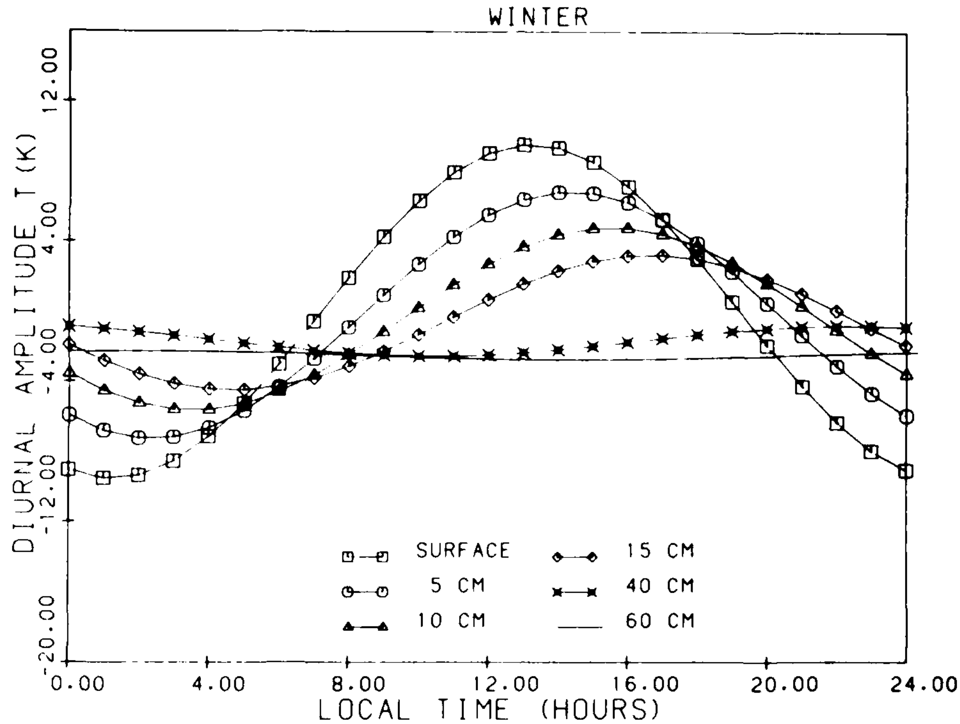

Figure 2. Model Diurnal Temperature Deviation (From the Surface Daily Mean) at Various Depths (Winter Case). Solar midnight is represented as 0.00 local time

$$\tilde{D} = I'/[(b + \lambda\beta)\cos\alpha + \lambda\beta \sin\alpha]. \tag{19b}$$

Recalling that the parameter b represents effects associated with the transient transfer of thermal radiation, sensible heat and latent heat, we see that the phase lag is determined principally by the ratio between the soil thermal capacity and the heat transfer associated with the feedback processes--the smaller the heat capacity and the more efficient the heat transfer at the ground, the smaller the phase lag. Similarly, the diurnal amplitude is determined by the balance between the solar flux I' and the non-solar heat fluxes at the ground--for a given I', the more efficient the transfer, the smaller the amplitude.

A comparison of the magnitude of the feedback terms in Eqs. (18b) and (19b) is given in Table 4. [See Eq. (12b) for definition of b.] For summer conditions, the amplitude and phase lag are affected mainly by latent and sensible transient heat fluxes; for typical winter conditions when $\partial q_s/\partial T$ is small, they are dominated by sensible heat transfer. Note from Eq. (12b) that since the transient sensible heat flux depends on $\gamma \rho_a u_a C_{HO}$ only, it is not a function of $\overline{T}_g$ and is thus affected in this model more by u_a than by the thermal structure in the boundary layer. The net transfer associated with thermal radiation, although a cubic func-

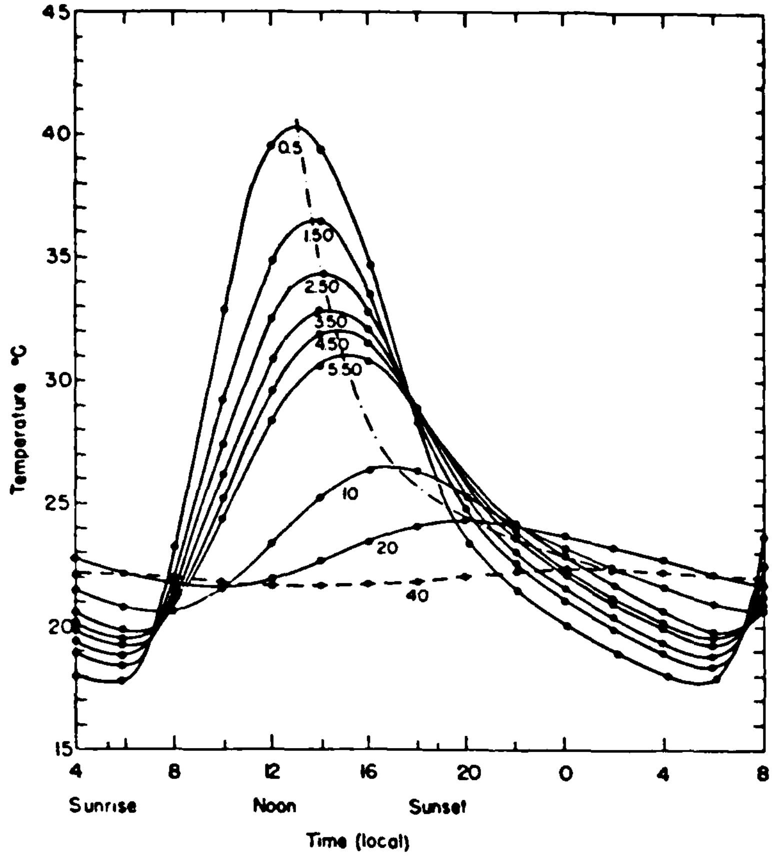

Figure 3. Measured Average Diurnal Soil Temperature Variations at Various Depths, O'Neil, Nebraska (After Kuo[10]). Depths are in cm

Table 3. Relative Magnitude of Various Components in the Clear-Sky Mean Surface Energy Balance Equation (W m^{-2})

	Summer	Winter
$\overline{I}$	352	145
$\overline{I}_L$	272	203
$-\overline{I}_g$	-439	-319
$-\overline{H}_s$	-25.1	16.7
$-\overline{LE}$	-145	-36.7
$-G_{soil}$	-14.9	-9.2

Table 4. Coefficients for the Various Feedback Terms in the Surface Energy Balance Equation (W m^{-2} K^{-1})

Feedback Term	Summer	Winter
longwave rad	4.65	3.62
sensible heat	11.9	17.0
latent heat	17.2	5.78
λE	13.3	13.3

tion of $\overline{T}$, is relatively small. We emphasize here that whether there is a transient heat gain or heat loss at the ground depends on the sign of D. For $D > 0$, heat is transferred from the ground surface to the atmosphere; for $D < 0$, the opposite is true. Thus, the net effect of the feedback is negative. That is, the transfer processes tend to smooth out the diurnal variations ather than to amplify them.

Thus, from an analysis of the feedback mechanism, we may identify the contribution of various physical processes to diurnal temperature variations within a soil slab. The larger the value of $\overline{T}_g$, the more important is the effect of latent heat transfer, provided that u_a is not zero and that a sufficient moisture supply exists. At low values of $\overline{T}_g$, latent heat transfer becomes less important.

To isolate the effect of latent heat transfer, we recomputed numerical values for Eqs. (13) and (20a) for the two cases given in Table 2, only this time omitting terms representing latent heat. Sample solutions are given graphically in Figures 4 and 5. As expected, both phase lags and amplitudes are larger than the corresponding cases shown in Figures 1 and 2. As a comparison, values of ϵ and $\hat{D}$ are listed in Table 5. We see that the inclusion of latent heat transfer is important for the summer case, reducing the surface mean temperature by 6 K, the phase lag by almost an hour and the amplitude by 6 K. In the winter case, its effect is less important.

6. DISCUSSION

Applying the perturbation method, we have derived an algebraic expression for the time variation of temperature within a model soil slab that is resting on a substrate of constant temperature and is heated from above at the upper surface. Evaluated at the upper surface, the expression gives diurnal temperature variations at the Earth's surface. This model differs from the classical model given by Blackadar[3] in that phase shift between the heating and the feedback terms in the

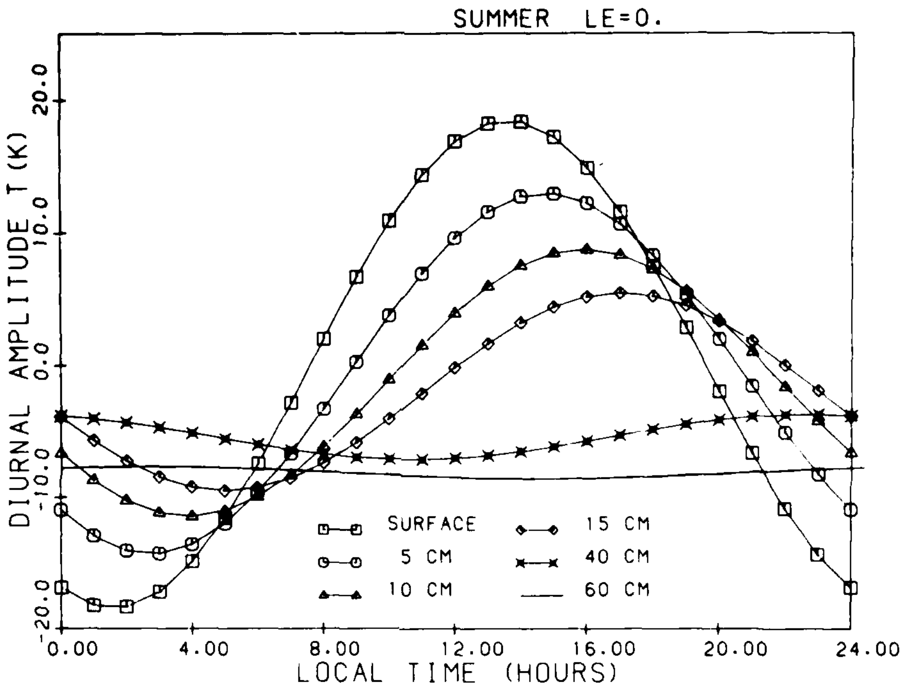

Figure 4. Model Diurnal Temperature Deviation (From the Surface Daily Mean) at Various Depths (Summer Case) but With No Evaporational Cooling at Ground Surface

upper boundary condition is permitted. As a result, the phase angle of the soil heat flux is allowed to differ from that of the solar heating. In fact, Eq. (10) states that the soil heat flux at the ground is determined by three components: (1) a time-invariant component represented by $\overline{F}$; (2) the diurnal component of solar heating, $I' \cos \omega t$; and (3) the linearized feedback bD due to the diurnal variation of ground and air temperatures. Note that the last two terms are not constrained to be in phase with each other.

Although phase shift in the feedback terms is permitted, the only external periodic forcing in our model is still limited to the $I' \cos \omega t$ term. As a result, our solution for T_g is also a function only of a single component cosine wave in t. However, if an external forcing with periods which are sub-multiples of period P is imposed on the upper boundary, the model solution will then possess composite waves. For example, if terms involving I_L, T_a, and q_a in Eq. (2) could be represented by a Fourier cosine series:

$$I_L + \rho C_p T_a + \gamma L A_m q_a = \sum_{k=0}^{K} f_k \cos k\omega t, \qquad (23)$$

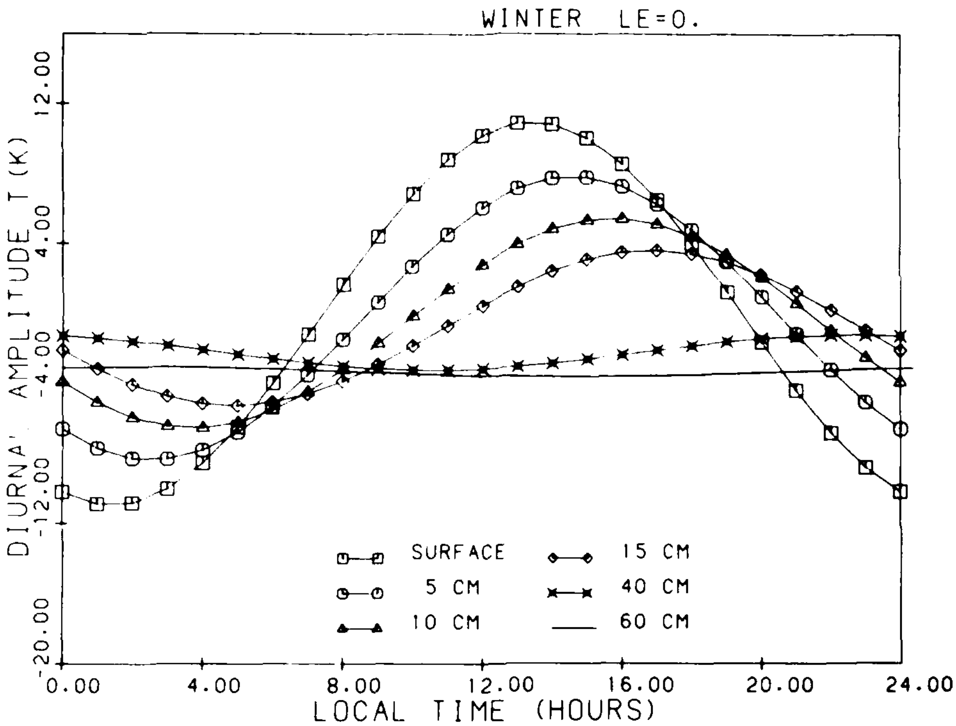

Figure 5. Model Diurnal Temperature Deviation (From the Surface Daily Mean) at Various Depths (Winter Case) but With No Evaporational Cooling at Ground Surface

Table 5. Effect of Surface Evaporation for Summer and Winter Cases

	Summer		Winter	
	Evap ≠ 0	Evap = 0	Evap ≠ 0	Evap = 0
$\overline{T}_g$ (K)	300.0	306.0	277.0	278.0
b (W $m^{-2}K^{-1}$)	33.7	16.8	26.4	20.7
α (deg)	15.8	23.8	18.5	21.4
D (K)	12.4	18.4	9.54	11.0

then the upper boundary condition for our model would become

$$\lambda \frac{\partial D}{\partial z} = \overline{F}_0 + \sum_{k=0}^{K} f_k \cos k\omega t + I' \cos\omega t - b_0 D, \tag{24}$$

where

$$\overline{F}_0 = \overline{I} - \varepsilon\sigma \overline{T}_g^{\,4} - \gamma C_p \overline{T}_g - \gamma L A_m q_s(\overline{T}_g), \tag{25}$$

$$b_0 = 4\varepsilon\sigma \overline{T}_g^3 + \gamma C_p + \gamma L A_m (\partial q_s/\partial T)_{\overline{T}g}, \tag{26}$$

$$f_k = \frac{C}{P} \int_0^P (I_L + \gamma C_p T_a + \gamma L A_m q_a) \cos k\omega t \, dt, \; k = 0, \ldots, K. \tag{27}$$

In Eq. (27), $C = 1$ if $k = 0$ or K; $C = 2$ otherwise. In practice, if this soil model is coupled with an NWP model, the period P in Eq. (27) could be defined as the 24-hr period preceding the current time-step. The important point here is that Eq. (24) is linear in D and, therefore, Eq. (9) may be solved in terms of a cosine series. In fact, the solution is simply the linear sum of the K + 1 independent solutions, each satisfying one of the K + 1 components in Eq. (24):

$$D = \sum_{k=1}^{K} \tilde{D}_k \exp(\beta_k z) \cos(k\omega t - \alpha_k + \beta_k z)$$
$$+ [C_0 + \sum_{k=1}^{K} \tilde{D}_k C_k \cos(k\omega t - \alpha_k + \beta_k d)] z. \tag{28}$$

Here the constants are determined by imposing Eqs. (24) and (11), and then equating coefficients for each of the components. In our model, we have ignored short term fluctuations of I_s and I_L associated with changes in cloudiness. Such a simplification enables us to retain only the $k = 1$ component in Eq. (28), which is given as Eq. (13).

Finally, it should be noted that this model may also be used to provide an analytical expression for the soil heat flux term used in the force-restore method. In this method, the prediction equation for the temperature of a slab with volumetric heat capacity C_s is given by

$$C_s \int_d^0 \frac{\partial T}{\partial t} \, dz = I_s + I_L - I_g - H_s - LE - \lambda \left(\frac{\partial T}{\partial z}\right)_d . \tag{29}$$

We may thus replace $(\partial T/\partial z)_d$ in Eq. (29) with $(\partial D/\partial z)_d$ obtained from Eq. (14). Essentially all we have done is to approximate the deep soil heat flux in the soil-slab model by that given in a linearized soil-slab model.

In summary, we have formulated an analytical soil-slab model that may be used to provide approximate values for the diurnal variation of ground surface temperature and to determine numerical values for the soil heat flux term in the

force-restore method. Compared with the classical model given by Blackadar,[3] ours has the following additional features:

(1) Phase shift is allowed between forcing and feedback terms.

(2) An explicit expression is given for the diurnal amplitude of the ground surface temperature. Although T_g given in this model is still represented only by a single-component cosine wave in t, it may be represented by a composite of waves in cases where the atmospheric forcing I_L, T_a, and q_a are known functions of time for the entire period P.

(3) Since soil temperature is given explicitly, an analytical expression for the soil heat flux term may be obtained simply by differentiating Eq. (13) and then evaluating at $z = d$.

As it stands, this model also permits us to isolate effects due to various physical processes, each represented by a different term in Eqs. (10), (12a), and (12b). This feature enables us to conduct numerical sensitivity studies with ease. For example, the response of diurnal amplitude and phase shift to moisture availability, surface roughness, and thermal capacity can be studied easily by changing the values of the coefficients of individual terms in Eqs. (10), (12a), and (12b).

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Glossary

a_0 = $S\bar{\tau}$ (1 - A) sin ϕ sin δ

a_1 = $S\bar{\tau}$ (1 - A) cos ϕ cos δ

A = planetary albedo

A_m = moisture availability

C_{HO} = heat transfer coefficient, valid for $z < z_a$

C_p = heat capacity of air at constant pressure

d = depth to bottom of soil slab

H_s = sensible heat flux

I_g = thermal flux from the ground

I_L = thermal flux from the atmosphere

I_s = $a_0 + a_1 \cos \omega t$, the solar flux

L = latent heat of evaporation

LE = latent heat flux

q_a = specific humidity of air at z_a

S = 1370 J m^{-2} sec^{-1}, the solar constant

t = time

T = temperature

T_a = temperature at z_a

T_d = temperature at infinite depth

T_g = temperature at ground surface

u_a = wind speed at z_a

z = distance from surface of ground

δ = solar declination

ε = emission coefficient for the ground

ε_a = emission coefficient for the atmosphere

λ = thermal conductivity of the soil

ρ_a = air density at a small distance z_a above the ground

σ = 5.67×10^{-8} J m^{-2} K^{-4} sec^{-1}, Stefan-Boltzmann constant

$\bar{\tau}$ = daily mean atmospheric transmissivity

Φ = latitude

ωt = local hour angle

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